P-67. A New Approach to Stereoscopic Display Developed by Solving the General Equation of Light Polarization

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Abstract: It is described a new approach to stereoscopic displays using amplitude modulation and elliptical polarization. The theory is developed by deriving and solving the general equation for elliptical polarization. Several transfer functions for novel stereoscopic displays are shown.

Key Words: 3D Stereo, 3D Imaging, stereoscopic display, elliptical polarization

1 Introduction

The dominance of LCD displays makes it desirable to use them for stereoscopic imaging. It is currently not possible to use ordinary LCD displays with the common time sequential methods using passive polarized or active LCD shutter glasses due to their slow response times. Consequently, it is desirable to develop such methods which use each *mn*–th pixel of such and LCD light modulation matrix (with M lines and N columns) for simultaneous presenting both *mn*–th elements L^{mn} and R^{mn} of left L and right R vews of 3D scene (Fig.**1**). Then the spatial resolution of 3D stereo image will be equal to full resolution MxN of the matrix.

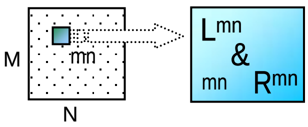

Fig.1 Joint mn-th elements of L&R

If we use both amplitude and polarization modulation it is possible to separate L from R with polarization decoding .

Thus, we present a general approach to stereoscopic LCD displays by solving the equation of light polarization. The general principle of this approach is described in [1].

2 Theoretical Approach

Set B_L^{mn} and B_R^{mn} as intensities of *mn*-th elements of left L and right R views of the initial 3D scene and J_L^{mn} and J_R^{mn} as intensities of left L and right R views that reach the eyes. In stage **1** the *mn*-th pixel of matrix display (Fig.**2**) creates an elementary light flux with intensity J_o^{mn} proportional to the sum of elementary light flux intensities of left J_L^{mn} and right J_R^{mn} views

$$J_o^{mn} = J_L^{mn} + J_R^{mn} = B_L^{mn} + B_R^{mn} . \quad (1)$$

In stage **2** the polarization of light of the same *mn*-th pixel. will be modulated by a coding algorithm that will allow separation by polarization decoding means **3**) of light flux J_o^{mn} of the two elementary light flux intensities J_L^{mn} and J_R^{mn}, which are directed to left and right eyes of the observer. To develop a new approach it is necessary to find the function for proper polarization encoding in *mn*-th pixel **2**. We express

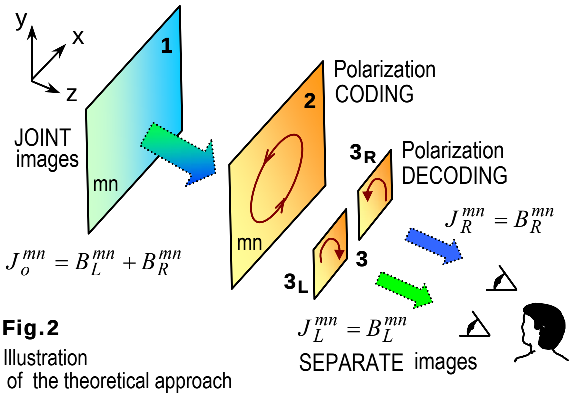

Fig.2 Illustration of the theoretical approach

the result of polarization encoding-decoding action as (2)

$$J_L^{mn} / J_R^{mn} = B_L^{mn} / B_R^{mn} . \quad (2)$$

Two surprisingly simple conditions (1) and (2) nevertheless provide the solution for the equation of polarization giving the desired polarization encoding function.

3 Derivation And Solving of The General Equation of Light Polarization

3.1 Deriving The Equation of Elliptical Polarization In The Required General Form

The electrical vector E of a harmonic plane light wave propagating along z axis is described by the expression

$$E^{mn} = E_o^{mn} \cos(\tau + \delta^{mn}), \quad (3)$$

where E_o is maximum amplitude of electrical vector, τ is the variable part of phase argument ($\tau = \omega t$ at fixed point of z), and δ is the initial phase shift. Determine E_x^{mn} and E_y^{mn} as x- and y-components of electrical vector E (Fig.**2**).

$$E_x^{mn} = E_o^{mn} \cos\varphi^{mn} \cos(\tau + \delta_1^{mn}), \quad (4)$$

$$E_x^{mn} = E_o^{mn} \cos\varphi^{mn} \cos(\tau + \delta_1^{mn}), \quad (4)$$

where angle φ is the initial angle of inclination of the electrical vector with respect to the x-axis, and δ_1^{mn} and δ_2^{mn} initial phase shifts of x- and y-components.

Eliminating common time factor τ from (1) and (2) by standard trigonometric transformations gives the desired general equation of elliptical light polarization

$$\frac{\left(E_x^{mn}\right)^2}{\cos^2\varphi^{mn}}+\frac{\left(E_y^{mn}\right)^2}{\sin^2\varphi^{mn}}-\frac{2E_x^{mn}E_y^{mn}}{\cos\varphi^{mn}\sin\varphi^{mn}}\cos\Delta\delta^{mn}=\left(E_o^{mn}\right)^2\sin^2\Delta\delta^{mn} \quad (6)$$

where $\Delta\delta^{mn}=\delta_1^{mn}-\delta_2^{mn}$ is the relative phase shift between x- and y-components.

3.2 Phase Shift Between X- And Y-Components

In case of a phase shift the equation of elliptical light polarization [1] takes the form

$$\left(E_x^{mn}\right)^2+\left(E_y^{mn}\right)^2-2E_x^{mn}E_y^{mn}\cos\delta^{mn}=\frac{(E_o^{mn})^2}{2}\sin^2\delta^{mn} \quad (7)$$

where E_o is amplitude of the electrical vector of light wave, E_x^{mn} and E_y^{mn} are x- and y- components of the electrical vector, and δ^{mn} is the phase shift between E_x^{mn} and E_y^{mn}.

The modulation parameter is the phase shift δ^{mn} whose value specifies the form of elliptical modulation.

Consider the case of fixed $\varphi^{mn}=\pi/4$ in (7). In this case it is necessary to use a *phase* modulator whose mn-th pixel is shown in the matrix modulator **2** (Fig.**1**). Equation (7) allows to find the transfer function for its mn-element by taking into account conditions (1) and (2). For example, suppose that polarization decoding is accomplished by mutually orthogonal polarization filters $\mathbf{3_L}$ and $\mathbf{3_R}$ disposed on x,y- coordinates (as in Fig.**3**). The direction of the polarization axis of filter $\mathbf{3_R}$ is described by y = x, and the direction of polarization of filter $\mathbf{3_L}$ by y = −x. The intensity of light from the polarization filter $\mathbf{3_R}$ is $J_{y=x}^{mn}$, and the intensity of light from the polarization filter $\mathbf{3_L}$ is $J_{y=-x}^{mn}$.

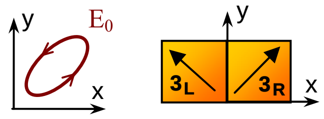

Fig.3 Analyzers with linear polarization

$\left(E_{x=y}^{mn}\right)^2$ is the light intensity $J_{x=y}^{mn}$ that is passing through the first linear analyzer ($y=x$)

$$J_{x=y}^{mn}=\frac{\left(E_O^{mn}\right)^2\sin^2\delta^{mn}}{4(1-\cos\delta^{mn})} \quad (8)$$

For the second analyzer ($y=-x$) substitution of corresponding $E_x^{mn}=-E_y^{mn}=E_{x=-y}^{mn}$ in (3) gives

$$J_{x=-y}^{mn}=\frac{\left(E_O^{mn}\right)^2\sin^2\delta^{mn}}{4(1+\cos\delta^{mn})} \quad (9)$$

Dividing (8) by (9) gives

$$\frac{J_{x=y}^{mn}}{J_{x=-y}}=\frac{J_L^{mn}}{J_R^{mn}}=\frac{1+\cos\delta^{mn}}{1-\cos\delta^{mn}} \quad (10)$$

From (5) and taking into account conditions (2) we have

$$\frac{B_R^{mn}}{B_L^{mn}}=\frac{1+\cos\delta^{mn}}{1-\cos\delta^{mn}} \quad (11)$$

from which follows the *desired transfer function* (10??) *for phase shift – solution value* δ_s^{mn}.

$$\delta_S^{mn}=\arccos\left[\frac{B_R^{mn}-B_L^{mn}}{B_R^{mn}+B_L^{mn}}\right] \quad (12)$$

It is easy to verify that function (12??) gives the required separation of left B_L^{mn} and right B_R^{mn} elements of the 3D scene in the amplitudes of left J_L^{mn} and right J_R^{mn} views, given by this approach. Indeed substitution of (12) in (11) gives

$$\frac{J_R^{mn}}{J_L^{mn}}=\frac{1+\cos\left[\arccos\left(\frac{B_R^{mn}-B_L^{mn}}{B_R^{mn}+B_R^{mn}}\right)\right]}{1-\cos\left[\arccos\left(\frac{B_R^{mn}-B_L^{mn}}{B_R^{mn}+B_L^{mn}}\right)\right]}=\frac{1+\left(\frac{B_R^{mn}-B_L^{mn}}{B_R^{mn}+B_L^{mn}}\right)}{1-\left(\frac{B_R^{mn}-B_L^{mn}}{B_R^{mn}+B_L^{mn}}\right)}=\frac{B_R^{mn}}{B_L^{mn}} \quad (13)$$

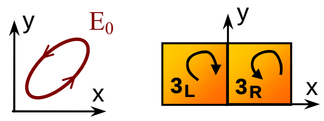

Fig.4 Analyzers with circular polarization

Taking into account condition (1) it is clear that the separation condition $J_L^{mn}=B_L^{mn}$ and $J_R^{mn}=B_R^{mn}$ is satisfied.

It can be supposed that using quadrature value $\delta_S^{mn}\pm k\frac{\pi}{2}$ of the phase shift (where k=1,2,..) permits the use of circular polalarization filters $\mathbf{3_L}$ and $\mathbf{3_R}$ (Fig.**4**) for separating $J_L^{mn}=B_L^{mn}$ from $J_R^{mn}=B_R^{mn}$.

3.3 Rotation of Plane Polarization (Optical Activity)

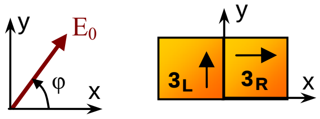

Fig.5 Analyzers with linear polarization

One case of elliptical polarization modulation is the rotation of linear polarization by angle φ (Fig.**5**). This effect is commonly called "optical activity" for substances performing such transformations of polarization. This case corresponds to the following notation of light polarization (at δ^{mn}=0)

$$\frac{\left(E_x^{mn}\right)^2}{\cos^2\varphi^{mn}}+\frac{\left(E_y^{mn}\right)^2}{\sin^2\varphi^{mn}}-\frac{2E_x^{mn}E_y^{mn}}{\cos\varphi^{mn}\sin\varphi^{mn}}=0 \quad (14)$$

Dividing (14) on $\left(E_x^{mn}\right)^2$ and multiplying on $\cos^2\varphi$ gives

$$\left(\frac{E_y^{mn}}{E_x^{mn}}\right)^2 - 2\left(\frac{E_y^{mn}}{E_x^{mn}}\right) tg\varphi^{mn} + tg^2\varphi^{mn} = 0 \qquad (15)$$

From (10) the *desired solution value* of φ_S^{mn} is

$$\varphi_S^{mn} = arctg\left(\frac{E_y^{mn}}{E_x^{mn}}\right) = arctg\left(\sqrt{\frac{J_y^{mn}}{J_x^{mn}}}\right) = actg\left(\sqrt{\frac{B_L^{mn}}{B_R^{mn}}}\right) \qquad (16)$$

3.4 Combining Phase Shift With Invariant Optical Activity

For combination (Fig.**6**) of phase shift δ^{mn} with invariant (relative to the form and the orientation angle φ^{mn} of elliptical polarization) optical activity of some hypothetical display leads to the general equation (6). For the first analyzer ($y = x$ direction) we have $E_x^{mn} = E_y^{mn} = E_{x=y}^{mn}$, substitution of which to (6) gives

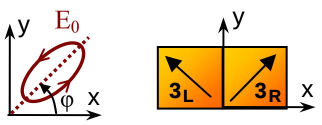

Fig.6 Analyzers with linear polarization

$$J_{x=y}\left(\frac{1-2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}{\cos^2\varphi^{mn}\sin^2\varphi^{mn}}\right) = \left(E_o^{mn}\right)^2 \sin^2\Delta\delta^{mn} \qquad (17)$$

and for the second analyzer ($y = -x$ directions) we have $E_x^{mn} = -E_y^{mn}$ (so $E_{x=-y}^{mn} = E_x^{mn} = -E_y^{mn}$)

$$J_{x=-y}\left(\frac{1+2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}{\cos^2\varphi^{mn}\sin^2\varphi^{mn}}\right) = \left(E_o^{mn}\right)^2 \sin^2\Delta\delta^{mn} \qquad (18)$$

Dividing (17) by (18) gives

$$\frac{J_{x=y}^{mn}}{J_{x=-y}^{mn}} = \frac{1+2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}{1-2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}} = \frac{B_R^{mn}}{B_L^{mn}} \qquad (19)$$

4 Realization of The Developed Approach With Liquid Crystal Matrices

Liquid crystal (LC) matrices of modern practical displays are polarization-based devices (as a rule, with a nematic LC layer) from which it is possible to designate two main groups: displays with electrically controlled birefringence (ECB), and those with electrically controlled optical activity (ECOA), i.e., a rotation of the linear polarization direction. The intensity modulation is accomplished by placing an LC matrix between two mutually crossed linear polarization films (usually called polarizer and analyzer). For polarization coding it is necessary to use an LC matrix without analyzer. The first case is the combination of the intensity matrix modulator with ECB matrix modulator (Fig. **7**) whose function is to make the phase shift $\Delta\delta^{mn}$ as a result of the difference between indices of refraction n_e and n_o for extraordinary *o* and ordinary *e* rays.

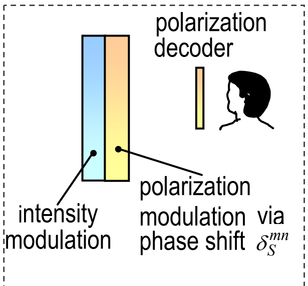

Fig.7 Combination modulator with ECB modulator

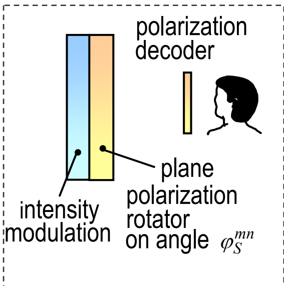

Fig.8 Combination intensity modulator with ECOA modulator

The second case is the combination of the intensity matrix modulator with ECOA matrix modulator (Fig. **8**). The possibility of using an LC matrix in such configuration is described in [2] and practical implementation in [3]. In both cases the observer views stereo images with help of passive polarized glasses (with linear or circular polarizing films) but it is also possible to have glasses-free viewing with, for example, a method of moving the boundary between two complementary optical layers [4].

4.1 Polarization Coding Birefringent LC Layer

A birefringent LC medium (Fig.**9**) creates an electrically controlled value of phase shift $\Delta\delta^{mn}$ between extraordinary *e* and ordinary *o* light rays (appearing inside the medium) that changes the elliptical polarization of the combined light ray (sum of ordinary and extraordinary rays).

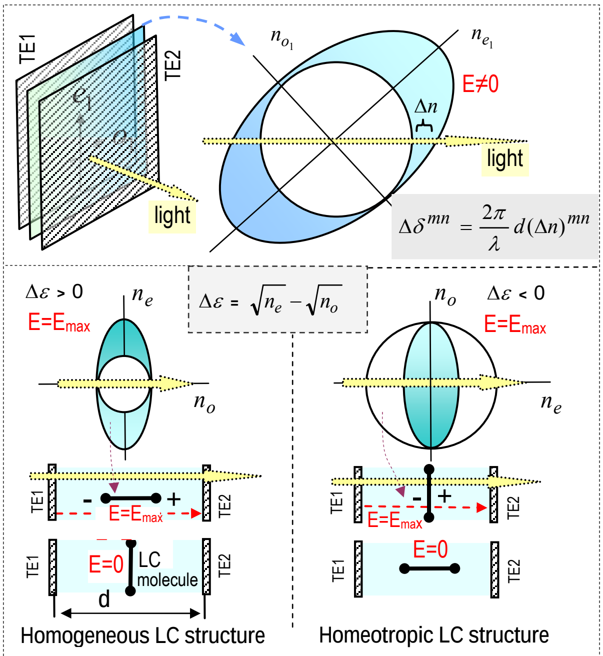

Fig.9 Creating phase shift $\Delta\delta^{mn}$ by controlled birefringence in LC

Such mediums include, for example:

- LC π cell and its modifications such as surface-mode LC cell (both are based on homogeneous LC structure with positive dielectric anisotropy $\Delta\varepsilon$),
- VA (Vertical Alignment) mode LC cell (which is based on homeotropic LC structure with negative dielectric anisotropy $\Delta\varepsilon$).

In both cases of LC with positive and negative anisotropy $\Delta\varepsilon$ the optical axis of uniaxial nematic LC layer is always parallel with the "length" of nematic LC molecule that looks like a "pencil" (Fig.**9**). Electric field E is applied to the LC layer via transparent electrodes (TE's). The birefringent LC cells with positive $\Delta\varepsilon$ are usually constructed so that at $E=E_{max}$ the phase shift is about zero $\Delta\delta^{mn}=0$ according to $\Delta n=0$ (Fig.**9**, left) assuming that in first-order approximation there is no residual Δn. It permits the maximum value of contrast because the interaction of crossed input and output polarizers (output one is the decoding polarizer) achieves the highest value of $E=E_{max}$ This forces the alignment of all LC molecules (excluding only those near surfaces of TE's) along the direction of the field E giving the greatest uniformity of LC molecule orientation in the mode of suppression of light flux (i.e., the degree of the suppression determines the contrast). So at E=0 LC molecules with positive $\Delta\varepsilon$ are arranged in parallel with the surfaces of TE's (such LC structure is called homogeneous). The case of LC layer with negative $\Delta\varepsilon$ is the opposite situation and the LC layer is referred to as homeotropic (Fig.**9**, right).

4.2 Polarization Coding LC Layer With Optical Activity

Optical activity of the LC layer creates the electrically controlled rotation of the plane polarization. As a rule, the angular direction (in plane of LC layer) of the plane polarization of input light must be fixed for correct function of the medium (i.e., optical activity of such medium can not be invariant relative to the angle of plane polarization of input light). Such medium include for example, a twisted LC cell with twist angle LC structure less than (or equal to) 90^0.

4.3 Polarization Coding LC Layer With Combined Birefringent And Optical Activity

Combination of birefringence with optical activity. There is a mixed group that consists of LC medium that combine ECB with ECOA. For example, there are supertwist LC cells with twist angle of LC structure more 90^0 (Fig.**10**). The intermediate states of light polarization are ellipsoids with different degrees of ellipticity at different angles of inclination.

The transfer function in this case is defined by formula (19).

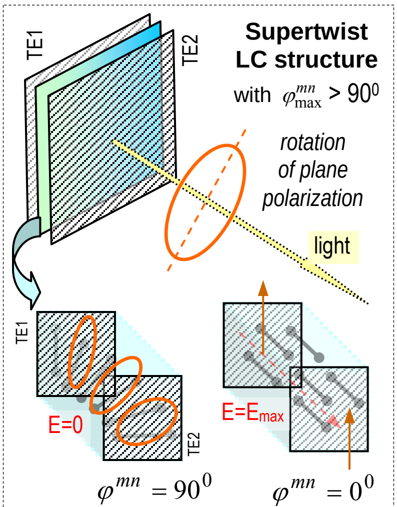

Fig.10 Combination of optical activity and birefringence in supertwist LC cells

4.4 Two Or More Separate Polarization Coding LC Layers

Two birefringent LC layers.

It is useful in this case to arrange two separate LC layers so that the ordinary ray of one LC layer will generate an extraordinary ray within another LC layer and vice versa. Such a configuration permits compensation of chromatic dispersion of both LC layers because only the extraordinary ray carries chromatic distortions. The corresponding transfer function of both LC layers is described by formula (12) by substituting the difference $\Delta\delta_S^{mn} = \Delta\delta_1^{mn}$ - $\Delta\delta_2^{mn}$ in the left side.

Two LC layers with optical activity.

Such a layout can work only if the second optically active LC layer (which receives light from the first optically active layer) possesses transverse angle-invariant optical activity. Such a requirement is not imposed on the first layer because it works at a fixed direction of plane polarization defined by the polarization axis of the preceding polarizer (i.e., of the analyzer of the preceding intensity modulating LC matrix). Also it is useful to employ mutually opposite directions of twist directions in the two LC layers in order to have maximum achromatic properties of the design as a whole. With proper choice of both optically active LC layers, the final transfer function is described by formula (16) if one places the difference $\varphi_s^{mn} = \varphi_1^{mn}$ - φ_2^{mn} in the left side.

Birefringent and optically active LC layers.

It is preferable to use the optically active LC layer as the first LC layer (in the direction of light flux) because in such case there is no necessity to be concerned with angle invariance for the optically active LC layer (i.e., the birefringent LC layer is invariant so it should be used as the second layer). The transfer function in this case is determined by formula (19).

Further investigations. As a continuation of this work there are also developed:

- world's first autostereoscopic 2D-3D switchable stereoscopic approach with FULL (MxN) resolution for EACH of L and R view [5],
- general decision of the described 3D imaging method allowing to use ANY type polarization-based LC matrixes characterized by arbitrary polarization effects (IPS, FFS, MVA, PVA, ASV…), including bistable ones (FLC…) [6].

5 Optional Observing Of Two Quite Different Monoscopic Images Each With Full Resolution

It is possible to observe simultaneously two quite different (i.e., not comprising a stereo pair) 2D images if one uses two different types of passive glasses each with the same states of polarization in both sides but with orthogonal directions (Fig.**11**). Such possibility exists due to the fact that there is no restriction on the content of the 2D images which are polarization coded according to the above equations.

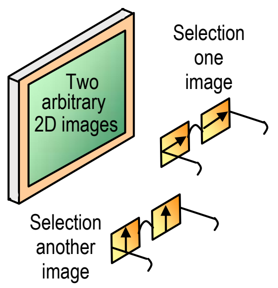

Fig.11 Simultaneous observing of two monoscopic images

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