Three-dimensional imaging with simultaneous reproduction of two image elements in one display pixel by information-dependent polarization coding

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Information-dependent (active) polarization encoding can be used to simultaneously present two image-resolvable elements [elements of left and right views of a three-dimensional (3D) scene] in a single display pixel. Polarization decoding, with the help of passive polarization filters, makes it possible to separate elements of left and right views and to observe them independently by left and right eyes. In this paper the basic theory of such 3D displays is developed. The relevant solutions of the general equation of light elliptical polarization are obtained in all important cases: cases of controlled birefringence and/or optical activity as three basic controlled polarization encoders. The obtained formulas are essentially the forms of signals that should control the values of birefringence and optical activity to realize the required polarization encoding. Optical schemes of flat-panel direct-view stereoscopic and autostereoscopic displays with the use of liquid crystal polarization encoding matrices are considered.

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1. Introduction

It is desirable to use a direct-view flat-panel display with Q pixels for simultaneously displaying two images, each having Q resolvable elements. This is especially useful for stereoscopic methods where a three-dimensional (3D) scene is represented by at least two (left and right) views observed simultaneously by left and right eyes.

Most modern flat-panel displays cannot be used for presenting two images using time sharing (frame sequential) methods due to long transient times. Lately, some fast 120 Hz liquid crystal (LC) displays (for example, Samsung Syncmaster 2233RZ) have been proposed to reproduce two views sequentially without a loss of spatial resolution. But such displays can be used only with active (shutter) glasses, which are heavier and require internal energy sources compared with easy passive glasses, and also the proposed display cannot be used without glasses (i.e., as a naked-eye or autostereoscopic display).

In this paper, we present a theoretical substantiation of stereoscopic and autostereoscopic displays where two image-resolvable elements of both (left and right) views are presented simultaneously at one pixel, with information-dependent elliptical polarization coding allowing the subsequent polarization decoding (with the help of polarization analyzers at the windows of the passive glasses or with the help of polarization parallax barriers–grids), providing separate left and right views by the corresponding eyes of the observer.

Also, such an approach can be used to simultaneously present two full-screen arbitrary images (not comprising the views of a definite 3D scene) for two viewers independently.

Practically, all LC displays are based on modulation of the light polarization state with subsequent

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polarization filtering to get the final intensity modulation for image formation. With such displays, the light polarization state can be modulated in addition to intensity values to double the information content of each pixel. Hence, standard LC matrices can be used in practice to realize theoretical methods, described in this paper, in the form of flat-panel direct-view 3D displays.

Let us first consider the rather simple basic conditions for simultaneously presenting two resolvable elements of different images in one display pixel for stereoscopic imaging.

2. Conditions for Separation of Two Views Presented by their Sum

Let the total number Q of display pixels be equal to $M \times N$, where M and N are numbers of rows and columns of the display matrix. Set B_L^{mn} and B_R^{mn} as luminance of the mnth resolvable elements of the left L and the right R views of the initial 3D scene, where $m = 1, 2, \ldots, M$ and $n = 1, 2, \ldots, N$. It is necessary to form two independent elementary light fluxes with intensities J_L^{mn} and J_R^{mn}, originating from the display screen, carrying information about left L^{mn} and right R^{mn} views:

$$J_L^{mn} \simeq B_L^{mn}; \qquad J_R^{mn} \simeq B_R^{mn}, \tag{1}$$

which should separately reach the corresponding (left and right) eyes of the observer. Proportionality constants in Eq. (1) are omitted for simplicity without loss of generality.

To satisfy Eq. (1), in the mnth pixel of light intensity matrix I of Fig. 1, it is necessary to create a common light flux with intensity J_o^{mn} equal to the sum of light fluxes with intensities J_L^{mn} and J_R^{mn} proportional to the sum of luminance B_L^{mn} and B_R^{mn}:

$$J_0^{mn} = J_L^{mn} + J_R^{mn} \simeq B_L^{mn} + B_R^{mn}. \tag{2}$$

In the mnth pixel of light polarization matrix E, the polarization of the common light flux J_o^{mn} is encoded according to an algorithm permitting subsequent decoding (via decoding left D_L and right D_R polarization filters) of two elementary light fluxes with intensities J_L^{mn} and J_R^{mn}, presenting luminance B_L^{mn} and B_R^{mn}, according to Eq. (1). Simultaneously maintaining the

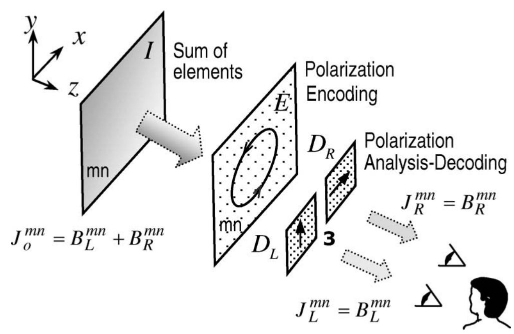

Fig. 1. Separation of simultaneous resolvable elements of two views by polarization coding.

two conditions [(1) and (2)] leads to the following condition for separating B_L^{mn} from B_R^{mn}:

$$J_L^{mn}/J_R^{mn} = B_L^{mn}/B_R^{mn}. \tag{3}$$

Simple expressions (2) and (3) further serve as boundary conditions to derive nontrivial solutions of the general equation of light polarization, which makes it possible to determine the forms of controlling signals for polarization encoding elements [1,2].

3. Elliptical Light Polarization in the Case of Combined Birefringence and Optical Activity

A. Controlled Optically Anisotropic Plates

A combined optically anisotropic plate [Fig. 2(a)] has united properties of the phase retarder **Ret** (ideal birefringent plate in the form of a uniaxial crystal) and an optically active plate (the ideal optical rotator **Rot**).

The birefringent plate **Ret** [Fig. 2(b)] has the birefringence effect only. The birefringence of the **Ret** is caused by the difference Δn between the refractive indices n_o and n_e ($\Delta n = n_o - n_e$) for ordinary and extraordinary rays propagating along axis o–o' and axis e–e', respectively. A corresponding relative phase shift $\Delta\delta$ between the E_y and the E_x components of light exists due to their differing speeds of propagation inside the plate along the o–o' and e–e' axes; hence, $\Delta\delta = \delta_o - \delta_e$, where δ_o and δ_e are the separate phase shifts for ordinary and extraordinary rays. The resulting phase retardation is $\Delta\delta = (2\pi/\lambda)\Delta n \times d$, where d is the physical thickness of the birefringent plate, and λ is the light wavelength.

The optical activity plate **Rot** [Fig. 2(c)] has the optical rotation effect only. The optical rotator **Rot** with optical activity $\Delta\varphi$ changes the angle of the linear polarization state of the input light wave on

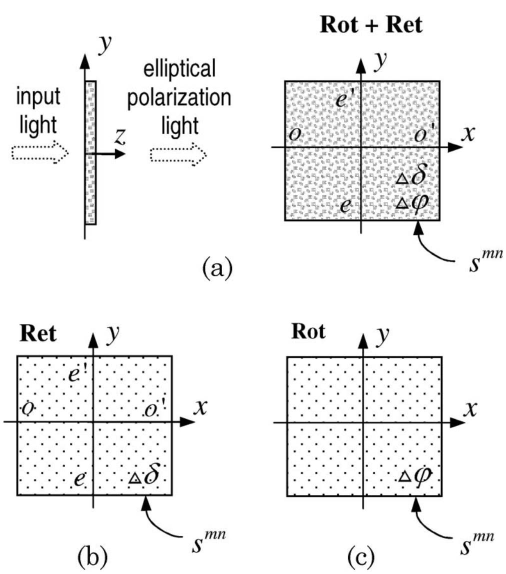

Fig. 2. Voltage-controlled optically anisotropic plate: (a) united phase retarder–optical rotator, (b) pure phase retarder, (c) pure optical rotator.

the $\Delta\varphi$ value or, in the general case, rotates the input arbitrary polarization shape on the same $\Delta\varphi$ value without changing the form of the shape.

The values of $\Delta\delta$ and $\Delta\varphi$ are controlled by the amplitude of external signal s to realize the required polarization encoding functions, and the corresponding controllable components will be designated as $\Delta\delta_S$ and $\Delta\varphi_s$.

B. General Equation of Light Polarization

We derive the general equation of light polarization corresponding to the combination of two basic effects of optical anisotropy–birefringence and optical activity.

For the mnth pixel, the electrical vector E^{mn} of an harmonic plane light wave, propagating along the z axis inside the combined anisotropic plate **Ret** + **Rot**, is described by the expression

$$E^{mn} = E_o^{mn} \cos(\tau + \delta^{mn}), \tag{4}$$

where E_0^{mn} is the maximum amplitude of the electrical vector, τ is the variable part of the phase argument ($\tau = \omega t$ at fixed point of z), and δ^{mn} is the initial phase factor. As the plane wave is a transverse field, only x and y components (E_x^{mn} and E_y^{mn}) of electrical vector E^{mn} are different from zero. The curve described by the end point of electrical vector E^{mn} is the locus of the points with the coordinates E_x^{mn} and E_y^{mn}:

$$E_x^{mn} = E_o^{mn} \cos\varphi^{mn} \cos(\tau + \delta_o^{mn}), \tag{5}$$

$$E_y^{mn} = E_o^{mn} \sin\varphi^{mn} \cos(\tau + \delta_e^{mn}), \tag{6}$$

where angle φ^{mn} is the resulting angle of optical rotation of electrical vector E^{mn} with respect to the x axis, and δ_o^{mn} and δ_e^{mn} are the phase shifts of x and y components, corresponding to ordinary n_o and extraordinary n_e refraction indices. Values $E_o^{mn} \cos\varphi^{mn}$ and $E_o^{mn} \sin\varphi^{mn}$ determine the projections of instantaneous values of electrical vector E^{mn} on the x and y axes, which are not contributed to by phase shifts δ_o^{mn} and δ_e^{mn} but caused by optical activity φ^{mn} only.

Eliminating the common time factor τ from Eqs. (5) and (6) by standard trigonometric transformations (analogous to ones executed in the classic polarization equation [3]) gives the desired general equation of light polarization:

$$\frac{(E_x^{mn})^2}{\cos^2\varphi^{mn}} + \frac{(E_y^{mn})^2}{\sin^2\varphi^{mn}} - \frac{2E_x^{mn}E_y^{mn}}{\cos\varphi^{mn}\sin\varphi^{mn}} \cos_\Delta \delta^{mn} = (E_o^{mn})^2 \sin^2\Delta\delta^{mn}, \tag{7}$$

where $\Delta\delta^{mn} = \delta_o^{mn} - \delta_e^{mn}$ is the relative phase shift between E_x^{mn} and E_y^{mn}.

According to Eq. (7), the light wave that passed through the combined anisotropic plate is elliptically polarized—the end of the electrical vector E^{mn} describes the ellipse (Fig. 3). The ellipse is inscribed into a rectangle whose sides are parallel to coordinate axes x and y. The projections of the ellipse circumference on coordinate axes x and y are equal to the corresponding projections of the diagonal of the rectangle. This diagonal corresponds to the length of the equivalent linear polarization electrical vector E_{lin}^{mn}, whose projections on the coordinate axes are equal to the corresponding projections of the polarization ellipse. The equivalent vector E_{lin}^{mn} makes angle φ^{mn} (corresponding to an equivalent optical rotation) with axis x; hence, the x projection of this vector is equal to the result of the pure optical activity (at zero value of phase shift $\Delta\delta^{mn}$) on the polarization state projections. The presence of nonzero phase shift $\Delta\delta^{mn}$ transforms the linear polarization vector into an elliptical polarization, with the $\Delta\varphi^{mn} - \Delta\delta^{mn}$ relationship expressed by the following equation:

$$\tan 2\alpha^{mn} = \tan 2\varphi^{mn} \cos\Delta\delta^{mn}, \tag{8}$$

which is derived from Eq. (7) by standard trigonometric transformations. The angle α^{mn} is made by the major (long symmetry) axis a–a' of the polarization ellipse with the x axis. The absence of the birefringence effect ($\Delta\delta^{mn} = 0$) gives $\alpha^{mn} = \varphi^{mn}$, corresponding to a degeneration of the ellipse into a linear polarization vector E_{lin}^{mn}, whose angle $\varphi_{\text{lin}}^{mn}$ with respect to the x axis coincides with angle α_{lin}^{mn} of the degenerate major axis.

The controlled birefringence $\Delta\delta^{mn}$ and optical activity φ^{mn} will be further considered (where it is necessary) as a sum of the variable (controllable) and constant components:

$$\Delta\delta^{mn} = \Delta\delta_S^{mn} + \delta_{\text{const}}^{mn}, \tag{9}$$

$$\varphi^{mn} = \Delta\varphi_S^{mn} + \varphi_{\text{const}}^{mn}, \tag{10}$$

where $\Delta\varphi_S^{mn}$ and $\Delta\delta_S^{mn}$ are the increment of optical activity angle and the increment of phase shift. Each of these are controlled by a signal s^{mn}, with the

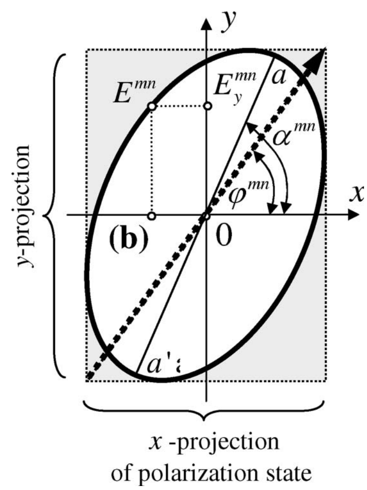

Fig. 3. Elliptical polarization dependence on phase retardation and optical activity.

purpose of fulfilling the desired polarization encoding function, and $\varphi^{mn}_{\text{const}}$ and $\Delta\delta^{mn}_{\text{const}}$ correspond to the initial conditions of the input light wave, which will be used elsewhere for obtaining alternative expressions for encoding functions.

4. Polarization Encoding Functions and Decoding Polarization Filters

Let us next find three sets of polarization encoding functions, corresponding to the three above-mentioned kinds of optically anisotropic plates.

A. Combination of Controlled Phase Shift and Controlled Optical Activity

The combination of phase shift $\Delta\delta^{mn}$ with optical rotation $\Delta\varphi^{mn}$ [Fig. 2(a)] is conveyed by general Eq. (7), and the desired solutions (encoding functions) will be obtained after defining the features of the polarization decoding filter (polarization analyzer), D.

First choose pair D_1 of the polarization decoding filters (Fig. 4) in the form of two mutually orthogonal linear polarization analyzers whose axes are directed along the bisectors $y = x$ and $y = -x$ of the angles between coordinate axes x, y. For the first polarization filter (the $y = x$ direction), we have the condition $E^{mn}_x = E^{mn}_y = E^{mn}_{x=y}$; the substitution of this into Eq. (7) gives

$$J^{mn}_{y=x}\left(\frac{1 - 2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}{\cos^2\varphi^{mn}\sin^2\varphi^{mn}}\right) = (E^{mn}_o)^2\sin^2\Delta\delta^{mn}, \quad (11)$$

where $J^{mn}_{y=x} = (E^{mn}_{y=x})^2$. For the second polarization filter ($y = -x$ direction), we have another condition: $E^{mn}_{x=-y} = E^{mn}_y = -E^{mn}_x$; the substitution of this into Eq. (7) gives

$$J^{mn}_{y=-x}\left(\frac{1 + 2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}{\cos^2\varphi^{mn}\sin^2\varphi^{mn}}\right) = (E^{mn}_o)^2\sin^2\Delta\delta^{mn}, \quad (12)$$

where $J^{mn}_{y=-x} = (E^{mn}_{y=-x})^2$.

The left and right eyes of the observer are located behind the corresponding polarization filters $D_{1(L)}$ and $D_{1(R)}$; hence, $J^{mn}_{y=x} = J^{mn}_L$ and $J^{mn}_{y=-x} = J^{mn}_R$. To meet condition (3), divide Eq. (11) by Eq. (12) to obtain

$$\frac{J^{mn}_{y=x}}{J^{mn}_{y=-x}} = \frac{J^{mn}_L}{J^{mn}_R} = \frac{B^{mn}_L}{B^{mn}_R} = \frac{1 + 2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}{1 - 2\cos\varphi^{mn}\sin\varphi^{mn}\cos\Delta\delta^{mn}}, \quad (13)$$

and, after standard trigonometric transformations,

$$\frac{J^{mn}_L}{J^{mn}_R} = \frac{B^{mn}_L}{B^{mn}_R} = \frac{1 + \sin 2\varphi^{mn}\cos\Delta\delta^{mn}}{1 - \sin 2\varphi^{mn}\cos\Delta\delta^{mn}}. \quad (14)$$

Expression (14) will provide values either of φ^{mn} (while using $\Delta\delta^{mn}$ as a parameter) or values of $\Delta\delta^{mn}$ (using φ^{mn} as a parameter) as functions of B^{mn}_L and B^{mn}_R. Calculated values of φ^{mn} and $\Delta\delta^{mn}$ are the desired sets of functions for proper polarization encoding.

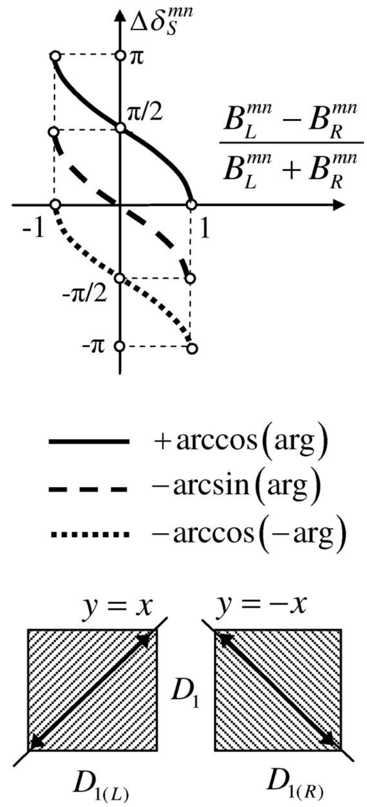

Fig. 4. Encoding functions in the case of voltage-controlled phase retardation.

It is difficult to use in practice the common analytic solution [(14)] for polarization coding because it is necessary to know the relationship between $\Delta\delta^{mn}_s$ and $\Delta\varphi^{mn}_S$ in the combined anisotropic plate with the fluctuation of signal s as an argument. The more simple analytic solutions take place in cases of separate nonzero $\Delta\delta^{mn}_s$ or $\Delta\varphi^{mn}_S$.

B. Controlled Phase Shift in the Case of Controlled Phase Shift Only

In the absence of controlled optical rotation, corresponding to $\Delta\varphi^{mn}_s = 0$ in Eq. (10), the anisotropic plate becomes a pure birefringent or a pure retarder plate [Fig. 2(b)]. Choose the initial angle $\varphi^{mn}_{\text{const}} = 45°$ to set equal conditions for the creation of both ordinary and extraordinary rays and substitute the resulting φ^{mn} from expression (10) into Eq. (14), receiving

$$\frac{B^{mn}_L}{B^{mn}_R} = \frac{1 + \cos\Delta\delta^{mn}_s}{1 - \cos\Delta\delta^{mn}_s}, \quad (15)$$

from which the desired solution (encoding function) for controlled phase shift $\Delta\delta^{mn}_s$ follows;

$$\Delta\delta^{mn}_s = \arccos\left[\frac{B^{mn}_L - B^{mn}_R}{B^{mn}_L + B^{mn}_R}\right]. \quad (16)$$

Variation of the controllable phase shift $\Delta\delta_s^{mn}$ (Fig. 4, solid curve) has the limits $[+\pi, 0]$ at appropriate limits $[-1, 1]$ of its argument fluctuations, corresponding to boundary values $B_L^{mn} = 0$ and $B_R^{mn} = 0$. At $\Delta\delta_s^{mn} = \pi$, the luminance of the left view B_L^{mn} is zero; hence, only partial light flux $J_R^{mn} = J_{y=-x}^{mn} = B_R^{mn}$ exists and goes to the right eye of the observer. Analogously, at $\Delta\delta_s^{mn} = 0$ (corresponding to $B_R^{mn} = 0$), only the partial light flux $J_L^{mn} = J_{y=x}^{mn} = B_L^{mn}$ exists and goes to the left eye of the observer. At intermediate values of B_L^{mn} and B_R^{mn}, corresponding intermediate values of $\Delta\delta_s^{mn}$ lead to proportional distribution of the light energy between the values of J_L^{mn} and J_R^{mn}.

Consider the case of the constant phase shift $\delta_{\text{const}}^{mn} = \pi$ in addition to the value of $\Delta\delta_s^{mn}$; substituting $\Delta\delta_s^{mn} + \pi$ into Eq. (16) gives

$$\Delta\delta_s^{mn} = -\arccos\left[\frac{B_R^{mn} - B_L^{mn}}{B_L^{mn} + B_R^{mn}}\right]. \tag{17}$$

In this case (Fig. 4, dotted curve), the controllable phase shift $\Delta\delta_s^{mn}$ has the limits $[0, -\pi]$ at appropriate limits $[-1, 1]$ of its argument fluctuations, but the total phase shift $\Delta\delta^{mn}$ has the limits $[+\pi, 0]$, simulating the behavior of the "+ arccos" encoding function by the sum of $\Delta\delta_s^{mn}$ with constant phase shift $\delta_{\text{const}}^{mn} = \pi$.

Consider the case of another constant phase shift $\Delta\delta_{\text{const}}^{mn} = \pi/2$, additional to the $\Delta\delta_s^{mn}$ value; substitution of $\Delta\delta_s^{mn} + \pi/2$ into Eq. (16) gives

$$\Delta\delta_s^{mn} = -\arcsin\left[\frac{B_L^{mn} - B_R^{mn}}{B_L^{mn} + B_R^{mn}}\right]. \tag{18}$$

Variation of the controllable phase shift $\Delta\delta_s^{mn}$ has the limits $[+\pi/2, -\pi/2]$ (Fig. 4, dashed curve), but the total phase shift $\Delta\delta^{mn}$ has the same limits $[0, +\pi]$ as in the two previous cases.

Linear polarization filters with mutually orthogonal axes are necessary for proper decoding of encoded functions determined by Eqs. (16)–(18) because the summary phase shift $\Delta\delta^{mn}$ has boundary values π and zero, taking into account the additional (to the controllable phase shift $\Delta\delta_s^{mn}$) constant phase shift $\Delta\delta_{\text{const}}^{mn}$. Circular polarization filters with mutually opposite directions of rotation of linear polarization can also be used if $\Delta\delta_{\text{const}}^{mn}$ is chosen such that boundary values of the summary phase shift $\Delta\delta^{mn}$ will be $\pm\pi/2$. If the views B_L^{mn} and B_R^{mn} are interchanged in Eqs. (16)–(18), then the linear or circular polarization filters should be interchanged, correspondingly, to preserve the proper polarity of the left and right views.

C. Controlled Optical Rotation in the Case of Controlled Optical Activity Only

In the absence of controlled phase shift, corresponding to condition $\Delta\delta_S^{mn} = 0$ in Eq. (9), the anisotropic plate becomes a pure optical action or an optical rotation plate [Fig. 2(c)]. At first we set $\Delta\delta_{\text{const}}^{mn} = 0$ also; hence, substitution of resulting $\Delta\delta^{mn} = 0$ from Eq. (9) into Eq. (14) leads to

$$\frac{B_L^{mn}}{B_R^{mn}} = \frac{1 + \sin 2\varphi^{mn}}{1 - \sin 2\varphi^{mn}}. \tag{19}$$

It is also worthwhile to obtain another version of Eq. (19), corresponding to using an additional constant angle $\varphi_{\text{const}}^{mn} = 90°$ in Eq. (10); hence, Eq. (19) is transformed to

$$\frac{B_L^{mn}}{B_R^{mn}} = \frac{1 + \sin[2(\varphi^{mn} + 90°)]}{1 - \sin[2(\varphi^{mn} + 90°)]} = \frac{1 - \sin 2\varphi^{mn}}{1 + \sin 2\varphi^{mn}}. \tag{20}$$

In this paper, concrete values of angle φ^{mn} are written in degrees for convenience, whereas the values of δ^{mn} are still expressed in radians. Eqs. (19) and (20) give the corresponding solutions:

$$\varphi_s^{mn} = +\frac{1}{2}\arcsin\left[\frac{B_L^{mn} - B_R^{mn}}{B_R^{mn} + B_{L:}^{mn}}\right], \tag{21}$$

$$\varphi_s^{mn} = -\frac{1}{2}\arcsin\left[\frac{B_L^{mn} - B_R^{mn}}{B_R^{mn} + B_{L:}^{mn}}\right], \tag{22}$$

whose plots are shown in Fig. 5. In the case of "$+1/2$ arcsin," the limits of each φ_s^{mn} and φ^{mn} are $[-45°, +45°]$. In the case of "$-1/2$ arcsin," the limits of φ_s^{mn} are $[+45°, -45°]$ and of φ^{mn} are $[+135°, +45°]$. In these cases, linear polarization filters D_2 with $\pm 45°$ orientation of axes are used.

The following two cases correspond to $\varphi_{\text{const}}^{mn} = \pm 45°$. In the case of a "+" sign, Eq. (19) takes the form

$$\frac{B_R^{mn}}{B_L^{mn}} = \frac{1 + \cos 2\varphi^{mn}}{1 - \cos 2\varphi^{mn}} = \tan^2\varphi. \tag{23}$$

In the case of a "–" sign, Eq. (19) gives

$$\frac{B_R^{mn}}{B_L^{mn}} = \frac{1 - \cos 2\varphi^{mn}}{1 + \cos 2\varphi^{mn}} = \cot^2\varphi. \tag{24}$$

Thus, the first corresponding solution is

$$\varphi_S^{mn} = \arctan\left(\frac{B_L^{mn}}{B_R^{mn}}\right)^{1/2}, \tag{25}$$

where at $\varphi_{\text{const}}^{mn} = +45^{\circ}$ the encoding function "arctan" [Fig. 6(a), solid line] and its argument have the limits:

$$\varphi^{mn} \in [0, 90^{\circ}]; \qquad \varphi_s^{mn} \in [-45^{\circ}, 45^{\circ}]; \qquad \left(\frac{B_L^{mn}}{B_R^{mn}}\right)^{1/2} \in [0, \infty]. \tag{26}$$

The second corresponding solution is

$$\varphi_S^{mn} = \operatorname{arccot}\left(\frac{B_L^{mn}}{B_R^{mn}}\right)^{1/2}, \tag{27}$$

where at $\varphi_{\text{const}}^{mn} = -45^{\circ}$ the encoding function "arccot" [Fig. 6(a), dotted line] and its argument have the following limits:

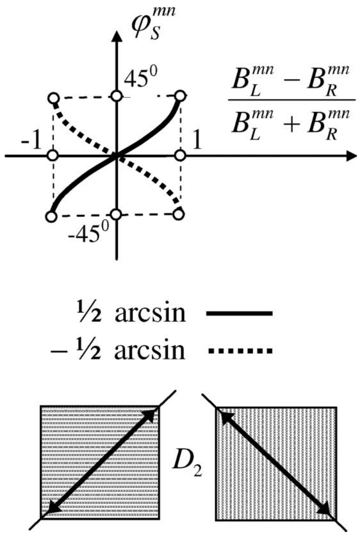

Fig. 5. Encoding functions in the case of voltage-controlled optical activity at ±45° angles between the input polarization direction and the polarization decoder axis.

$$\varphi^{mn} \in [90^\circ, 0]; \qquad \varphi^{mn} \in [135^\circ, 45^\circ]; \qquad \left(\frac{B_L^{mn}}{B_R^{mn}}\right)^{1/2} \in [0, \infty]. \tag{28}$$

The narrower parts of the plots (at negative values of the argument) are unused: only argument values within the interval $[0, \infty]$ have a physical sense. There is asymptotic behavior at one end of each plot. In this case, a pair D_3 of linear polarization filters is used.

The encoding function given by Eq. (25) agrees with the encoding function obtained in [4] with the help of Malus' formula for an optical rotator (in the form of a LC 90° twist structure) for stereo imaging with corresponding arrangement of two polarization analyzers.

Rotation invariance of the system with optical activity makes it possible to change the polarization decoding filter axes, in concert with changing the limits of values of encoding functions. For example, it is possible to use a pair D_4 of polarization filters [Fig. 6(b)], with axes directed along the x and y axes (Fig. 5), if $\varphi_{\mathrm{const}}^{mn} = 0^\circ$ and the encoding function φ_s^{mn} has limits $[0, 90^\circ]$ and $[90^\circ, 0]$ for "arctan" and "arccot," respectively.

5. Schemes For Stereoscopic and Autostereoscopic Imaging

Stereoscopic imaging with the use of derived encoding–decoding functions is possible with polarized glasses or for glasses-free (autostereoscopic, naked-eye) observation.

A. Stereoscopic Observation with Polarized Glasses

The mnth element of the light intensity matrix I is adjacent to the mnth element of the light polarization matrix E (Fig. 7). The signal s_Σ^{mn}, given on the electrical input of the light intensity matrix, corresponds to the sum of light intensities described by Eq. (2). The signal $s_{\Delta\delta}^{mn}$ (or s_φ^{mn}), given on the electrical input of the light polarization matrix E, realizes condition (3), at the expense of forming polarization encoding functions according to Eqs. (16)–(18), (21), (22), (25), and (27). These signals are obtained by processing the initial signal s (carrying information about any two views of a 3D scene) with the help of electronic processing units PU_1 and PU_2. The modulated common light flux is decoded by a pair D of polarization filters, which have mutually orthogonal polarization characteristics and are used as the lenses of the viewing glasses; hence, the left and right eyes of the observer perceive corresponding light intensities J_L^{mn} and J_R^{mn} [5].

To calculate the final form of the controlling signal s^{mn}, it is necessary to take into account the dependency of the values of $\triangle\delta_s^{mn}$ and $\triangle\varphi_s^{mn}$ on the amplitude of a signal s.

B. Glasses-Free (Autostereoscopic) Observation

Figure 8(a) shows a cross section of the autostereoscopic layout [6], corresponding to the mth row of intensity matrix I and polarization encoding matrix E, whose frontal views are shown in Fig. 8(b). The light intensity matrix I performs the same function as in the previously considered case, but the light polarization encoder matrix E has mutually orthogonal polarization states in adjacent columns, relative to the luminance B_L^{mn} and B_R^{mn}. One polarization state, for example, corresponds to odd $2n-1$ columns of

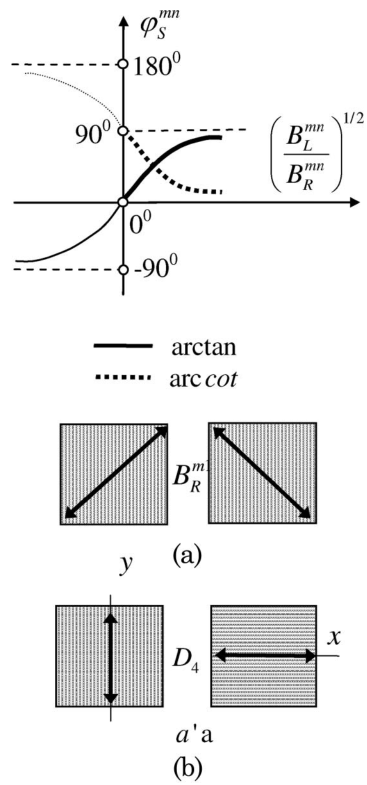

Fig. 6. Encoding functions in the case of voltage-controlled optical activity at 0° – 90° angles between the input polarization direction and the polarization decoder axis: (a) first orientation of axis and (b) second orientation of axis.

polarization encoder E and another (orthogonal to the first) polarization state corresponds to even $2n$ columns. Polarization decoder D comprises a columnar phase-polarization parallax barrier B [Fig. 8(b), right] and a polarization analyzer A. The columnar phase-polarization parallax barrier B consists of multiple phase shifters and/or optical rotators, which are organized in a static grid structure with a period similar to the period of disposition of columns in matrices I and E. Polarization analyzer A is placed behind phase-polarization parallax barrier B to transform its action on light polarization states into light intensity variations. The left and right spatially separated observing zones, Z_L and Z_R, are formed at a specific distance from the parallax barrier, providing glasses-free observation with full-screen resolution for each view. Such arrangement of polarization states allows one to obtain the same polarization state for all those partial light fluxes that reach both observation zones Z_L and Z_R. It leads to using one and the same (single) polarizer sheet as polarization analyzer A for both observation zones. The separation of left and right views is as follows. Light elliptical polarization, for example, from the odd pixel $m1$ of polarization encoder matrix E, is characterized by the value of the x component of polarization (projection of polarization state on axis x arranged in the plane of the drawing of Fig. 8), corresponding to the first pixel of the left view B_L^{m1} and by the value of the y component of polarization, corresponding to the even pixel $m2$ of the right view B_R^{m2}. Light elliptical polarization from the even pixel $m2$ is characterized by orthogonal (relative to the pixel $m1$) x and y components of polarization directions for B_L^{2m} and B_R^{2m}. Polarization decoder D forms partial light fluxes, corresponding to all luminance elements of left (right) views, having required horizontal (x direction) polarization, only for light paths leading to the left Z_L (right Z_R) observation zone; hence, only such partial light fluxes successfully pass through polarization analyzer A. However, all cross light fluxes (having the vertical direction of light polarization behind parallax barrier B) are rejected by polarization analyzer A.

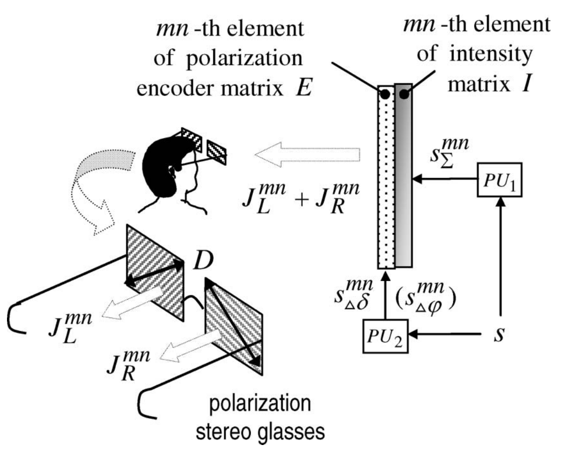

Fig. 7. Stereoscopic observation with glasses.

Such spatial filtering is performed analogously for each of the N rows of image-forming matrices.

Mutually orthogonal (relative to B_L^{mn} and B_R^{mn}) polarization states between adjacent columns of the polarization encoder E can be created by various means. One way is to interchange B_L^{mn} and B_R^{mn} in the arguments of functions given by the Eqs. (16)–(18), (21), (22), (25), and (27) to get the orthogonal polarization state, i.e., to interchange B_L^{mn} and B_R^{mn} in controlling signals on adjacent columns. This leads to mirroring the plots of these functions relative to the y axis on even columns in comparison with odd ones. For example, this is shown in Fig. 9 for a solid line plot as shown above in Fig. 3. Another way is to use additional phase shift in the even columns without interchanging B_L^{mn} and B_R^{mn}. This is mathematically equivalent to the first way:

$$\Delta\delta_S^{m(2n-1)} = \arccos\left(\frac{B_L^{m(2n-1)} - B_R^{m(2n-1)}}{B_L^{m(2n-1)} + B_R^{m(2n-1)}}\right), \qquad (29)$$

$$\Delta\delta_S^{m(2n)} = \arccos\left(-\frac{B_L^{m(2n)} - B_R^{m(2n))}}{B_L^{m(2n)} + B_R^{m(2n)}}\right) = \pi - \arccos\left(\frac{B_L^{m(2n)} - B_R^{m(2n))}}{B_L^{m(2n)} + B_R^{m(2n)}}\right). \qquad (30)$$

This way can be realized not only by action of electrical signal s, but by combination of the optical phase shift π (realized with help of optical π retarders in the even columns) with electrically controlled phase shift $\Delta\delta_S^{m(2n)}$.

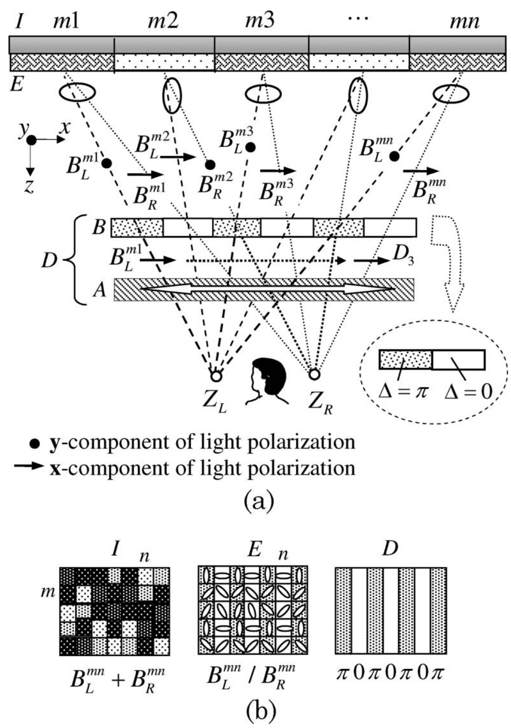

Fig. 8. Glasses-free (autostereoscopic) observation with switched phase grid.

In Fig. 10 an unswitched polarization decoder D was used in the form of a polarization grid P (with mutually orthogonal polarization directions in adjacent elements), which can be mechanically inserted in the scheme to view 3D images without glasses. Removing grid P from the scheme allows one to view 3D images with polarization glasses with a corresponding electrical switching of the polarization state of polarization matrix E to eliminate the presence of two mutual orthogonal polarization states in its columns (which is necessary only in the case of using polarization grid P). It is also possible to view monoscopic images with the naked eye. Possible small discrepancies in the longitudinal z position of polarization grid P do not destroy the two image zones Z_L and Z_R but can only shift them slightly along the z axis. Possible small discrepancies in the transverse x and y positions of polarization grid P practically do not influence the spatial positions of these zones because of the corresponding properties of the Fresnel spectrum of grid P (where these zones are formed). Only rotational movements of grid P are inadmissible.

C. Liquid Crystal Components as Polarization Encoders and Decoders

Practically all commercial LC displays use nematic LCs. Nematic LCs are characterized by rodlike polar molecules capable of being oriented by their long axes in parallel with, or orthogonally to, the lines of force of an external (controlling) electric field, depending on the positive or negative sign of the LC dielectric anisotropy $\Delta\varepsilon$, respectively, where $\Delta\varepsilon = \varepsilon_o - \varepsilon_e$ and ε_o and ε_e are dielectric constants of the LC layer for ordinary and extraordinary rays defined as $\varepsilon_o = (n_o)^{1/2}$ and $\varepsilon_e = (n_e)^{1/2}$. In the cases of $\Delta\varepsilon > 0$ or $\Delta\varepsilon < 0$, the initial orientation of LC molecules (their long axes) of the LC cell are selected accordingly to be parallel (homogeneous orientation) or orthogonal (homeotropic orientation) to the boundaries of the LC layer adjacent to the inner surfaces of two glass substrates, with the spacing interval between them defining the thickness d of the LC layer. Thin (several micrometers) nematic LC layers, with homogeneous or homeotropic orientations, correspond to anisotropic plates with voltage-controlled birefringence and/or optical activity. Changing the electro-optical properties of homogeneous or homeotropic LC layers by the electric field is accompanied by a splay or a bend deformation of LC layers called the S and B effect, correspondingly. A homogeneous LC cell with $\Delta n \times d = \pi$ and with the opposite signs of pretilt angles of surface LC molecules at opposite sides of the LC layer is named by the π cell [7], which is characterized by excellent transient time (no more than several milliseconds) and small chromatic dispersion in comparison with the classic S effect. Pretilt angles are the angles in the longitudinal z direction (elevation angles). A homeotropic LC cell is often called a vertically aligned (VA) mode cell. The π cell and VA mode cell are perfect electrically controlled birefringent plates (**Ret** plates) for the input light wave with an arbitrary elliptical polarization state.

A twisted nematic LC structure (T effect) [8] is characterized by different transverse angles (φ_1 and φ_2) of the initial orientation of LC surface molecules on the opposite sides of the LC layer with $\Delta\varepsilon > 0$. The twist angle in intermediate transverse planes inside the LC layer continuously varies from φ_1 to φ_2. The transverse (azimuth) angle corresponds to the angle in any plane parallel to the boundaries of the LC layer. Change of transverse optical rotation angle $\Delta\varphi_s = \varphi_1 - \varphi_2$ is caused by longitudinal rotation of the polar LC molecules under force of the external field induced by the controlling electrical signal s.

The 90° twist LC structure ($\Delta\varphi_s = 90^\circ$) is a perfect optical rotator of light linear polarization (**Rot** plate) if its direction makes a zero or 90° transverse angle ($\varphi_0 = 0^\circ$ or $\varphi_0 = 90^\circ$) with respect to the orientation of the surface LC molecules on the boundary of the LC layer, corresponding to the input light wave.

The LC twist cell with its own optical activity angle $\Delta\varphi_s \neq 90^\circ$ creates combined birefringent and optical rotation effects, even under conditions $\varphi_0 = 0^\circ$ or $\varphi_0 = 90^\circ$ for input linear polarization (so it can play the role of the **Rot** + **Ret** plate). An LC twist cell with its own 90° optical activity cannot work as a pure

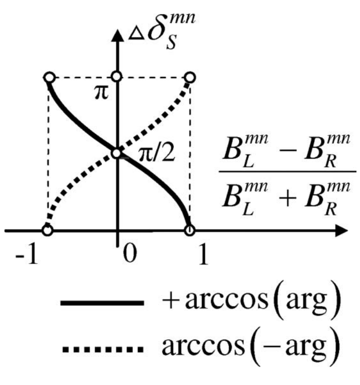

Fig. 9. Plots of two mutually orthogonal encoding functions.

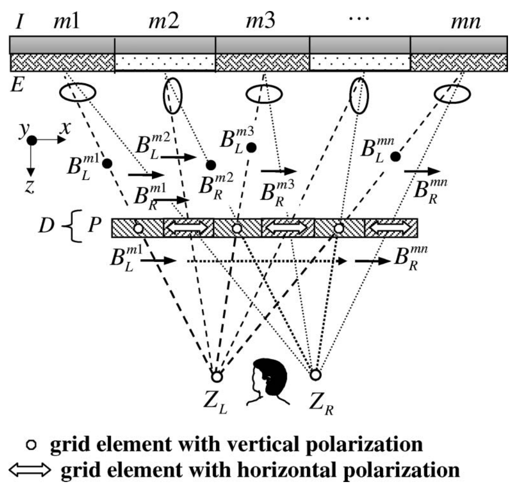

Fig. 10. Autostereoscopic observation with polarization grid.

optical rotator under conditions $\varphi_0 \neq 0^\circ$ or $\varphi_0 \neq 90^\circ$ for input linear polarization; hence, the output light wave has elliptical polarization. For the same reason, in the case of arbitrary elliptical polarization of the input light wave, the polarization state of light that passes through the 90° angle LC twist cell is not simply the rotated input polarization state shape, but another form of elliptical polarization as a result of the complex interaction of optical activity and birefringence.

The π cell, VA mode cell, and 90° twist cell can successfully work as polarization encoders. Polarization decoders can be made with π cell and VA mode cells because of their ability to introduce the correct phase shift $\Delta\delta_s$ at an arbitrary polarization state of the input light. However, it is problematic to use a twist cell (regardless the value of $\Delta\varphi_s$) as a polarization decoder according with this approach, as it is very difficult to calculate analytically the form of output polarization in the case of an arbitrary elliptical input polarization.

Controlling the electrical field inside the LC layer is done by electrical signals applied to transparent electrodes adjacent to both boundaries of the LC layer and adhered to the surrounding transparent (glass) substrates.

Any type of matrix display (LCD, DLP, OLED, PDP, etc.) can be used as a light intensity matrix I.

A practical 3D display using a 90° twist LC structure as a polarization encoding matrix and passive polarization glasses is described in [4], which works according to the algorithm defined by expressions (25)–(27).

Universal planar 3D displays providing full-screen resolution at each view can be made for use with passive (polarized) glasses (for many viewers) or without using any glasses (for a single viewer). Also such a 3D display can be used in 2D mode without loss of spatial resolution. One and the same display design can be used in three-pointed modes by electrical switching or by easy mechanical removal–insertion of the definite polarization-selective grid in a 3D display design.

6. Polarization Coding for Twin and Multiple Full-Resolution Imaging

In the above formulas, instead of luminance B_L^{mn} and B_R^{mn} of two views of the same 3D scene, it is possible to substitute luminance B_1^{mn} and B_2^{mn} of two quite different images. To separately observe these two images, glasses of two types should be used: the first one has analyzers with one polarization state in both windows, and the second type has analyzers with the second polarization state (orthogonal to the first one) in both windows. To obtain glasses-free observation, it is necessary to create a structure providing sufficient width in each observation zone for both eyes of each observer. One zone Z_1 (with the first image B_1^{mn}) will be for both eyes of the first observer, and another zone Z_2 (with the second image B_2^{mn}) will be for both eyes the second observer.

For example, to reproduce K images simultaneously, four images ($K = 4$) with elementary luminance B_1^{mn}, B_2^{mn}, B_3^{mn}, and B_4^{mn}, it is theoretically possible to use the following multistage procedure. One uses the common light flux with intensity $J_K^{mn} = J_{1+2}^{mn} + J_{3+4}^{mn}$, where $J_{1+2}^{mn} \simeq B_1^{mn} + B_2^{mn}$ and $J_{3+4}^{mn} \simeq B_3^{mn} + B_4^{mn}$, which is the multiple analog of formula (2) for double imaging. At the first stage, the polarization state of the common light flux is encoded according to the ratio $J_{1+2}^{mn}/J_{3+4}^{mn} \simeq (B_1^{mn} + B_2^{mn})/(B_3^{mn} + B_4^{mn})$, which is the multiple analog of formula (3). With the first polarization decoder, the common light flux is divided into two primary partial light fluxes, J_{1+2}^{mn} and J_{3+4}^{mn}. At the second stage, each of these partial light fluxes is divided into two secondary partial light fluxes with use of analogous encoding–decoding means, finally forming K light fluxes $J_K^{mn} = B_K^{mn}$ distributed in the corresponding zones of space.

The maximum dynamic range of each of K simultaneously reproduced images is equal to A/K, where A is the dynamic range of the light intensity matrix.

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